Effects of Self-Fields on Growth Rate in a Free-Electron Laser with Planar Wiggler Magnetic Field

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***Abstract*— The effects of self-fields on growth rate in a planar wiggler free-electron laser with axial magnetic field are investigated. It is shown that the growth rate decreases due to the self-fields for group I orbits. For group II orbits, the behavior of growth rate are interesting because it increases due to the self-fields.**

I. Introduction And Background

THE free-electron laser (FEL) is a high–intensity, continuously tunable source of coherent radiation [1]. The electromagnetic radiation is generated by the passage of a relativistic electron beam through a wiggler magnetic field. The electron beam can be collimated by a uniform static axial magnetic field produced by the current in a solenoid. It is known that the self–electric and self–magnetic fields induced by charge and current densities of the electron beam have significant effects on the operation of a free-electron laser [1]. Recently, more exact calculations of self–fields and analysis of their effects on electron dynamics in helical and planar wiggler FELs with axial magnetic field or ion-channel guiding have been published [2-5].

The purpose of the present paper is to study the effects of self-fields on dispersion relation and growth rate in a FEL with planar wiggler operated in the high gain regime. A uniform axial magnetic field is used, as a guiding field, to collimate the relativistic electron beam in the transverse direction and to enhance the growth rate [1]. In Sec. II dispersion relation showing the coupling of electromagnetic and space charge waves by the wiggler magnetic and ion-channel electrostatic fields in the presence of the self-fields is derived. In Sec. III, a numerical study of self-fields effects on growth rate is presented and discussed.

II. Dispersion Relation In The Presence of Self-Fields

Consider a relativistic electron moving along the z axis of a palanr wiggler magnetic field described by

$$\mathbf{B}_w = B_w \hat{\mathbf{e}}_y \sin k_w z \tag{1}$$

where B_w is the wiggler magnetic field amplitude and k_w is the wiggler wave number. The equation of motion of the electron in the presence of self-fields can be written as

$$\frac{d(m\gamma \mathbf{v})}{dt} = -e\left[\mathbf{E}_s + \frac{1}{c}\mathbf{v}\times(\mathbf{B}_w + B_0\hat{\mathbf{e}}_z + \mathbf{B}_s)\right] \tag{2}$$

where γ is the relativistic factor, c is the speed of light in vacuum,

$$\mathbf{E}_s = -2\pi e n_b (x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y) \tag{3}$$

is the self-electric field [5],

$$\mathbf{B}_s = 2\pi e n_b \beta_\parallel (y\hat{\mathbf{e}}_x - x\hat{\mathbf{e}}_y) + B_w\left[\lambda_x \cos(k_w z)\hat{e}_x + \lambda_y \sin(k_w z)\hat{e}_y\right] \tag{4}$$

is the self-magnetic field [5], $\beta_\parallel \equiv v_z / c$ is the normalized axial velocity,

$$\lambda_x \equiv \frac{2\omega_b^2 \Omega_0 \beta_\parallel^3}{\Omega_0^2\beta_\parallel^2 - [\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]^2}, \tag{5}$$

$$\lambda_y \equiv \frac{2\omega_b^2 \beta_\parallel^2[\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]}{\Omega_0^2\beta_\parallel^2 - [\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]^2}, \tag{6}$$

$\omega_b \equiv (2\pi e n_b / m\gamma k_w^2 c^2)^{1/2}$ is the normalized electron beam frequency and $\Omega_0 \equiv eB_0 / m\gamma k_w c^2$ is the normalized axial magnetic field.

The quasisteady-state solution for the normalized electron velocity components can be obtained in the form [5]

$$v_{xq} = \frac{c\Omega_w \beta_\parallel^2[\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]}{\Omega_0^2\beta_\parallel^2 - [\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]^2}\cos(k_w z), \tag{7}$$

$$v_{yq} = \frac{c\Omega_w \Omega_0 \beta_\parallel^3}{\Omega_0^2\beta_\parallel^2 - [\omega_b^2(1+\beta_\parallel^2)+\beta_\parallel^2]^2}\sin(k_w z), \tag{8}$$

$$v_{zq} = c\beta_{zq} \equiv c\beta_\parallel = \text{const.} \tag{9}$$

where $\Omega_w \equiv eB_w / m\gamma k_w c^2$ is the normalized wiggler magnetic field.

The dispersion equation can be obtained by using (7)-(9) and solving electron momentum transfer equation, the electron equation of continuity, Faraday's law, and Ampere-Maxwell law. This yields

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$$
\begin{aligned}
&\omega^2-k^2c^2-\omega_b^2\Bigg\{\frac{2(\omega-kc\beta_{\|})^2}{(\omega-kc\beta_{\|})^2+\omega_b^2\gamma_{\|}^{-2}-\Omega_0(\omega-kc\beta_{\|})}\\
&-\frac{1}{4}\frac{(k+k_w)(\alpha_x+\alpha_y)c}{[\omega-(k+k_w)c\beta_{\|}]^2-2\omega_b^2\gamma_{\|}^{-2}}\\
&\times\Bigg[(kc-\beta_{\|}\omega)(\alpha_x+\alpha_y)\\
&-\frac{\Omega_w(1+\lambda_y+\lambda_x)(\omega-kc\beta_{\|})^2}{(\omega-kc\beta_{\|})^2+\omega_b^2\gamma_{\|}^{-2}-\Omega_0(\omega-kc\beta_{\|})}\Bigg]\Bigg\}
\end{aligned}
\tag{10}
$$

Here ω is the electromagnetic (radiation) frequency and k is electromagnetic wave number.

III. Results And Discussion

The complex electromagnetic wave number k as a function of the electromagnetic wave frequency ω can be obtained by solving numerically the dispersion equation (10). This solution is used to study growth rate. For numerical calculations the normalized wiggler magnetic field Ω_w, the relativistic factor γ and the normalized electron beam frequency ω_b were taken to be 0.05, 6 and 0.2 respectively.

Graph of the normalized spatial growth rate $\mathrm{Im}(k)/k_w$ as a function of the normalized radiation frequency ω/ck_w is shown in Fig 1 for group I orbits. Graph of the normalized spatial growth rate in the absence of the self-fields is also shown in Fig. 1 for comparison. As shown in the figure the (maximum) growth rate in the presence of the self-fields is less than the growth rate in the absence of the self-fields. Therefore, the growth rate decreases due to the self-fields for group I orbits.

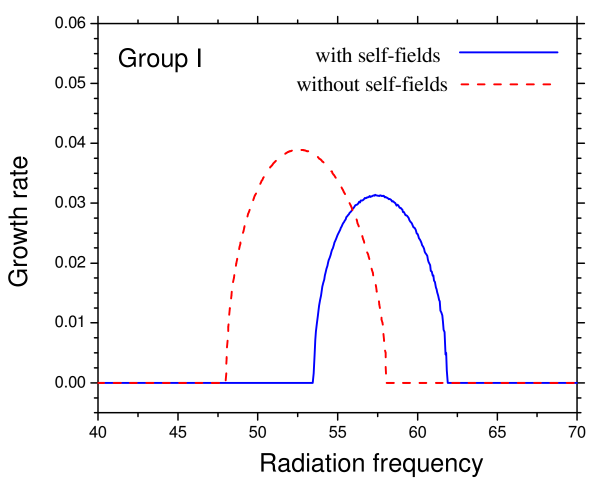

Fig. 2. Graph of the normalized growth rate $\mathrm{Im}(k)/k_w$ as a function of the normalized frequency ω/ck_w for group I orbits.

For group II orbits, graph of the normalized spatial growth rate $\mathrm{Im}(k)/k_w$ as a function of the normalized radiation frequency ω/ck_w is shown in Fig 2. Graph of the normalized spatial growth rate in the absence of the self-fields is also shown in Fig. 2 for comparison. As we see in this figure the (maximum) growth rate in the presence of the self-fields is greater than the growth rate in the absence of the self-fields. Hence, the growth rate enhancement is found due to the self-fields for group II orbits.

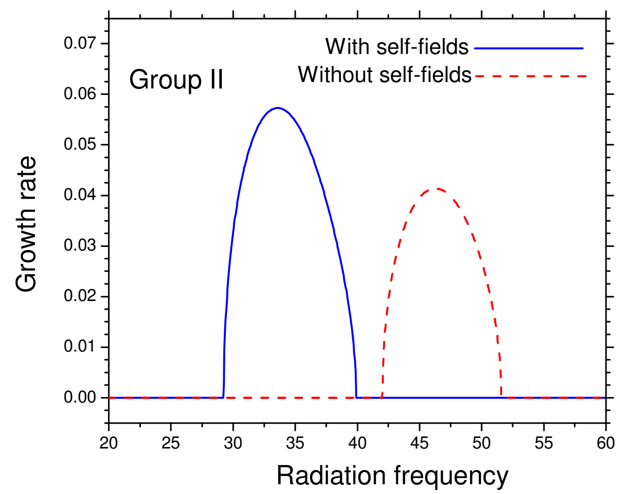

Fig. 2. Graph of the normalized growth rate $\mathrm{Im}(k)/k_w$ as a function of the normalized frequency ω/ck_w for group II orbits.

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